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OPTICAL VORTICES IN WAVEGUIDES WITH SPATIAL DEPENDENCE OF THE NONLINEAR REFRACTIVE INDEX

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ABSTRACT

In present work it is studied the formation of optical vortices in waveguides with spatial dependence of the nonlinear refractive index. The propagation of such type of laser pulses is governed by a system of amplitude equations for x and y components of the electrical field in which it is taken into account the effects of second order dispersion and self-phase modulation. The corresponding system of equations is solved analytically.

New class exact solutions, describing the generation of vortices structures in optical fibers with spatial dependence of the nonlinear refractive index and anomalous dispersion, are found. These vortices admit only amplitude type singularities. Their stability is a result of the balance between diffraction and nonlinearity, as well as nonlinearity and angular distribution. This kind of singularities can be observed as a depolarization of the vector field in the laser spot.

Keywords: *Optical vortices, vector amplitude equation, nonlinear refractive index*

INTRODUCTION

Optical vortex is referred to a beam or pulse that has singularity in the amplitude or phase. The last one is characterized by helical phase front. These light structures are solutions of two-dimensional paraxial scalar equation of Leontovich. They are usually created outside the laser cavity by using optical holograms and different optical masks.

The behavior of optical vortices in different waveguides is described by the nonlinear amplitude equation (3D+1 nonlinear Schrodinger equation) in which it is included a term, corresponding to the spatial dependence of the nonlinear refractive index (x^2+y^2). Amplitude modulations in such optical structures are observed in the case of studying the vector structure of the electric field and they are investigated in the frames of a system of two scalar nonlinear amplitude equations for the x and y components of the vector electric field.

A solution of the 3D+1 nonlinear Schrodinger equation for optical fibers with spatial dependence of the nonlinear refractive index was found for the first time by the authors in [1,2] and it was observed a formation of optical solitons.

Vortex structures as mode conversions have been observed recently in active resonators [3,4,5]. Optical vortices have a number of applications in the field of high resolution microscopy, optical tweezers, quantum information transfer, optical vortex trapping and many others.

BASIC THEORY

The equation describing the propagation of optical vortices in waveguides with spatial dependence of the nonlinear refractive index is in the form [1,2]:

$$-i\alpha\frac{\partial\vec{A}}{\partial z}+\frac{|\beta|}{2}\frac{\partial^2\vec{A}}{\partial t^2}+\frac{1}{2}\Delta_\perp\vec{A}+\gamma(x^2+y^2)\left|\vec{A}\right|^2\vec{A}=0, \tag{1}$$

where $\vec{A}$ is the vector amplitude function of the pulse envelope, t is time, α, β and γ are constants, characterizing respectively the number of oscillations under the pulse's envelope, dispersion and nonlinearity of the fiber, $\Delta_\perp$ is the transverse operator of Laplace. They are of the kind:

$$\alpha=k_0z_0,\ |\beta|=k_0u^2|k''|,\ \Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ \gamma=\frac{\alpha^2}{2}n_2|A_0|^2, \tag{2}$$

where k_0 *is* the wavenumber, z_0 is the initial longitudinal length, u, k'', n_2 are the group velocity, the second order of the linear dispersion and the nonlinear refractive index of the medium and A_0 is the magnitude of the initial amplitude.

BASIC THEORY

We have in mind that $\vec{A} = (A_x, A_y, 0)$ It is accepted that the axis Oz coincides with the geometrical axis of the fiber. Thus, we will work in cylindrical coordinates:

$$x = r\cos\theta, \quad r^2 = x^2 + y^2, \qquad \Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{3}$$

$$y = r\sin\theta, \quad \theta = arctg(y/x),$$

After couple of transformations, the equations describing the evolution of the components A_x and A_y of the vector $\vec{A}$, written in polar coordinates, can be presented as follow:

$$-i\alpha\frac{\partial A_x}{\partial z} + \frac{|\beta|}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial A_x}{\partial r} + \frac{\partial^2 A_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_x}{\partial \theta^2}\right) + \gamma r^2\left|A_x^2 + A_y^2\right|A_x = 0,$$

$$-i\alpha\frac{\partial A_y}{\partial z} + \frac{|\beta|}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial A_y}{\partial r} + \frac{\partial^2 A_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_y}{\partial \theta^2}\right) + \gamma r^2\left|A_x^2 + A_y^2\right|A_y = 0. \tag{4}$$

BASIC THEORY

In order to find a solutions of the equations above we make the following substitutions in the system of equations (4):

$$A_x(r,\theta,z,t) = P_x(r,\theta)e^{i(az+bt)},$$
$$A_y(r,\theta,z,t) = P_y(r,\theta)e^{i(az+bt)}, \qquad (5)$$

where a and b are constants, P_x and P_y are new unknown real functions. After couple of transformations we obtain:

$$(2\alpha a + b^2|\beta|) = \frac{1}{P_x}\left(\frac{1}{r}\frac{\partial P_x}{\partial r} + \frac{\partial^2 P_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_x}{\partial \theta^2}\right) + 2\gamma r^2\left|P_x^2 + P_y^2\right|,$$
$$(2\alpha a + b^2|\beta|) = \frac{1}{P_y}\left(\frac{1}{r}\frac{\partial P_y}{\partial r} + \frac{\partial^2 P_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_y}{\partial \theta^2}\right) + 2\gamma r^2\left|P_x^2 + P_y^2\right|. \qquad (6)$$

The left sides of the equations above are the same constant expressions. Their right sides are functions of the variables r and θ. In order to fulfill the equalities, we assume that:

$$2\alpha a + b^2 \,|\, \beta \,| = 0\,. \qquad (7)$$

BASIC THEORY

From this equality we find a connection between the constants *a* and *b*:

$$\alpha = -\frac{|\beta|}{2\alpha} b^2 \ . \tag{8}$$

Having in mind the expression (7), equations (6) take the form:

$$\begin{aligned} &\frac{1}{r}\frac{\partial P_x}{\partial r} + \frac{\partial^2 P_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_x}{\partial \theta^2} + 2\gamma r^2 \left| P_x^2 + P_y^2 \right| P_x = 0, \\ &\frac{1}{r}\frac{\partial P_y}{\partial r} + \frac{\partial^2 P_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_y}{\partial \theta^2} + 2\gamma r^2 \left| P_x^2 + P_y^2 \right| P_y = 0. \end{aligned} \tag{9}$$

We make another substitution:

$$\begin{aligned} &P_x(r,\theta) = R_x(r) e^{in\theta}, \\ &P_y(r,\theta) = R_y(r) e^{in\theta}, \quad n = const. \end{aligned} \tag{10}$$

BASIC THEORY

By using the substitution (10), the system of equations (9) can be presented in the form:

$$\begin{aligned}
&\frac{1}{r}\frac{\partial R_x}{\partial r}+\frac{\partial^2 R_x}{\partial r^2}-\frac{n^2}{r^2}R_x+2\gamma r^2(R_x^2+R_y^2)R_x=0,\\
&\frac{1}{r}\frac{\partial R_y}{\partial r}+\frac{\partial^2 R_y}{\partial r^2}-\frac{n^2}{r^2}R_y+2\gamma r^2(R_x^2+R_y^2)R_y=0.
\end{aligned} \tag{11}$$

Taking into account the nonlinear terms in equations (11), it is convenient to search for solutions of the kind:

$$\begin{aligned}
R_x&=Br^{\mu}\cos\left(mr^{\eta}\right),\\
R_y&=Br^{\mu}\sin\left(mr^{\eta}\right),
\end{aligned} \tag{12}$$

where B, m, μ, η are constants, about to be defined.

BASIC THEORY

By substituting expressions (12) in the system of differential equations (11) and after short transformations, we obtain:

$$\begin{aligned} & r^{\mu-2}\cos(mr^{\eta})\left[\mu^2-n^2-m^2\eta^2r^{2\eta}+2\gamma B^2r^{2\mu+4}\right]-m\eta r^{\mu+\eta-2}\sin(mr^{\eta})\left[2\mu+\eta\right]=0, \\ & r^{\mu-2}\sin(mr^{\eta})\left[\mu^2-n^2-m^2\eta^2r^{2\eta}+2\gamma B^2r^{2\mu+4}\right]-m\eta r^{\mu+\eta-2}\cos(mr^{\eta})\left[2\mu+\eta\right]=0, \end{aligned} \tag{13}$$

To fulfill the equalities in (31), it is needed that the coefficients in front of the respective trigonometric functions in both equations are zero. Thus, we obtain the following system of two ordinary differential equations:

$$\begin{aligned} & 2\mu+\eta=0, \\ & \mu^2-n^2-m^2\eta^2r^{2\eta}+2\gamma B^2r^{2\mu+4}=0. \end{aligned} \tag{14}$$

By using this system of equations we can define that:

$$\mu=-\frac{2}{3},\ \eta=\frac{4}{3},\ n^2=\mu^2,\ B=\frac{4m}{3\sqrt{2\gamma}}. \tag{15}$$

BASIC THEORY

Thus, the following exact analytical solutions, describing the optical vortices, propagating in fibers with spatial dependence of the nonlinear refractive index are found:

$$P_x = \frac{4m}{3\sqrt{2\gamma}} r^{-2/3} \cos\left(mr^{4/3}\right) e^{i\frac{2}{3}\theta},$$
$$P_y = \frac{4m}{3\sqrt{2\gamma}} r^{-2/3} \sin\left(mr^{4/3}\right) e^{i\frac{2}{3}\theta}, \quad m = 1,2,3... \tag{16}$$

Going back trough all the substitutions and assumptions made by now, the solutions for the components A_x and A_y of the vector $\vec{A}$amplitude function of the optical vortex, satisfying the basic equations (1) are of the kind:

$$A_x = \frac{4m}{3\sqrt{2\gamma}} r^{-2/3} \cos\left(mr^{4/3}\right) e^{i\frac{2}{3}\theta - i(az+bt)},$$
$$A_y = \frac{4m}{3\sqrt{2\gamma}} r^{-2/3} \sin\left(mr^{4/3}\right) e^{i\frac{2}{3}\theta - i(az+bt)}, \quad a = -\frac{|\beta|}{2\alpha} b^2, \quad m = 1,2,3... \tag{17}$$

GRAPHICS

- **Graphics of vortex solutions for *m=1***

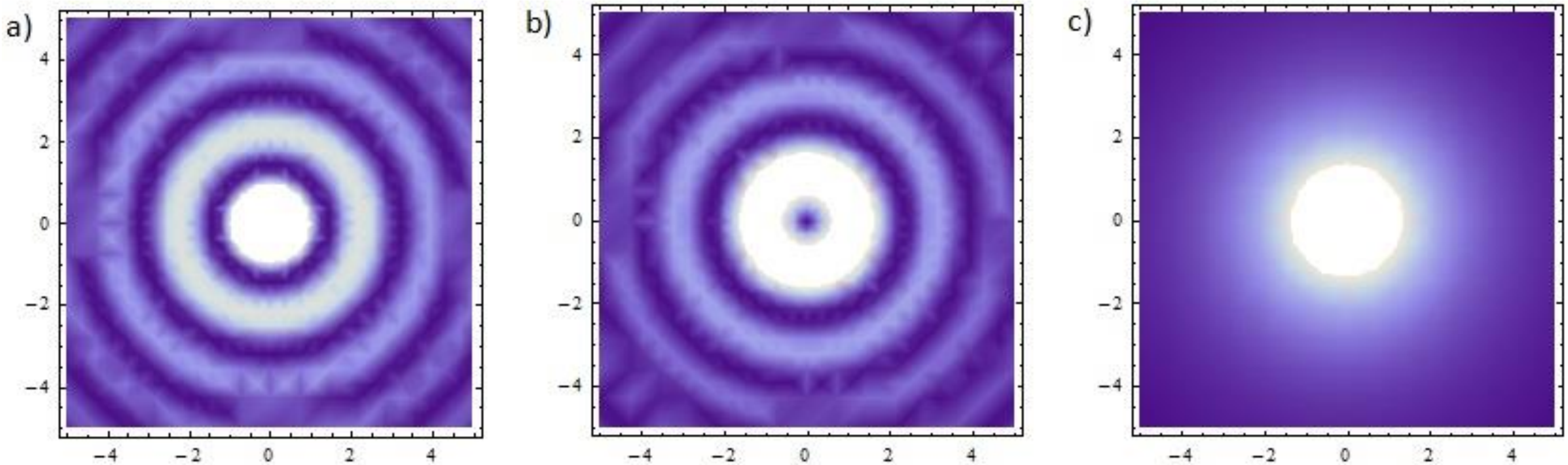

Fig 1. Intensity profiles of the components (a) A_x and (b) A_y and (c) the total intensity profile for m=1.

In Fig. 1 a) and b) it is presented the intensity profile of the vortex structure in the x and y components of the vector $\boldsymbol{A}$. In the total intensity profile $|A^2|$, vortex structures are not found (Fig. 1(c)).

GRAPHICS

- **Graphics of vortex solutions for *m=1***

The rotation of the vector ***A*** in the center of the optical vortex is shown in Fig. 2.

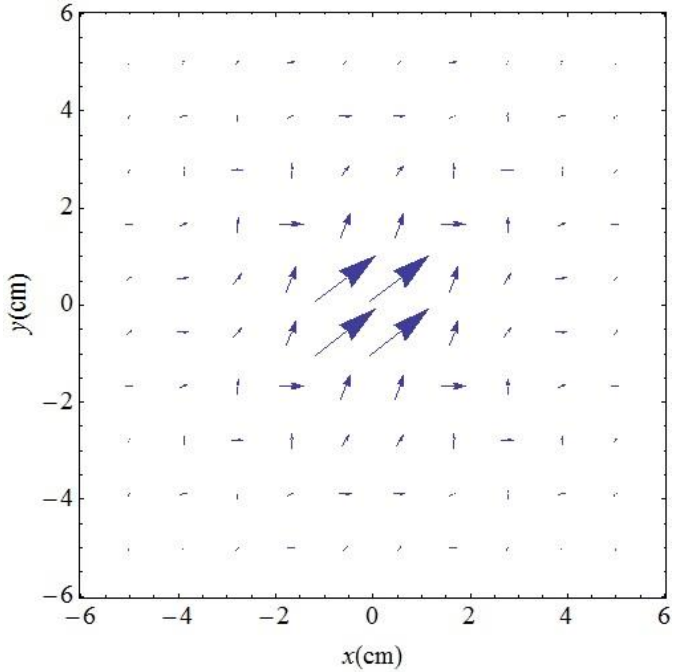

*Fig. 2 Diagram of the vector amplitude function for m=1. Significant rotation of the vector **A** in the center of the vortices is observed.*

GRAPHICS

- **Graphics of vortex solutions for *m=2***

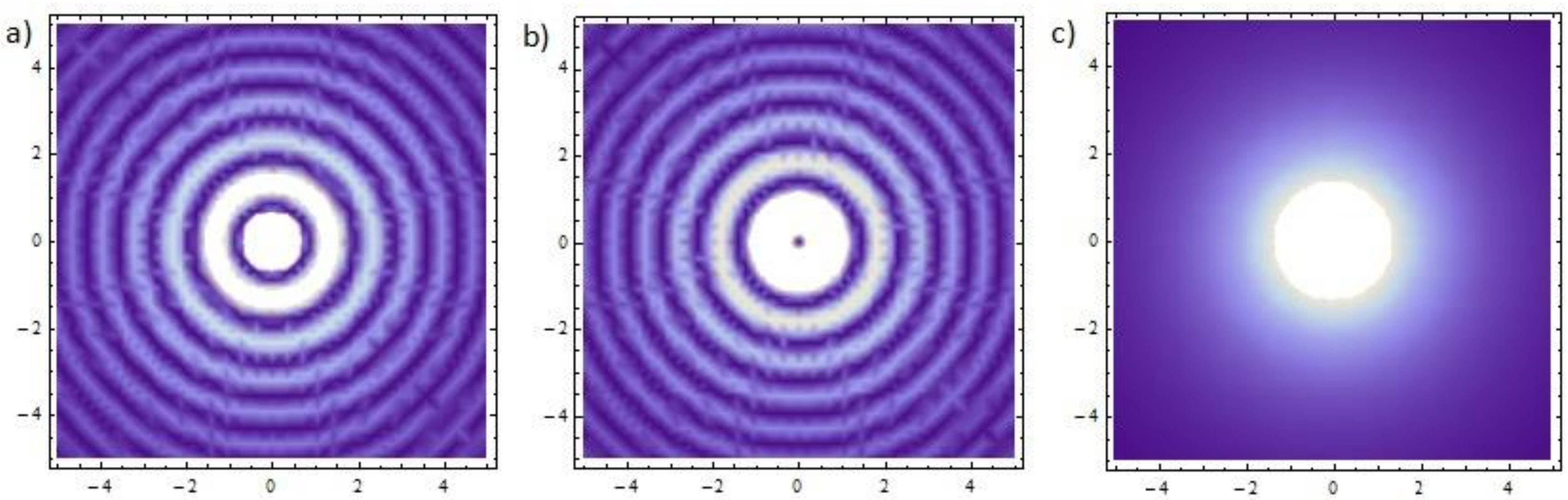

Fig. 3 Intensity profiles of the components (a) A_x , (b) A_y and (c) the total intensity profile for m=2.

In Fig. 3 it is presented the intensity profiles of the components *x* and *y* of the vector ***A***.

GRAPHICS

- **Graphics of vortex solutions for *m=2***

It is clearly seen that the increasing of the vortex parameter *m* leads to a significant change in vorticity and depolarization in the vector diagram (Fig. 4) for the case of *m=2*.

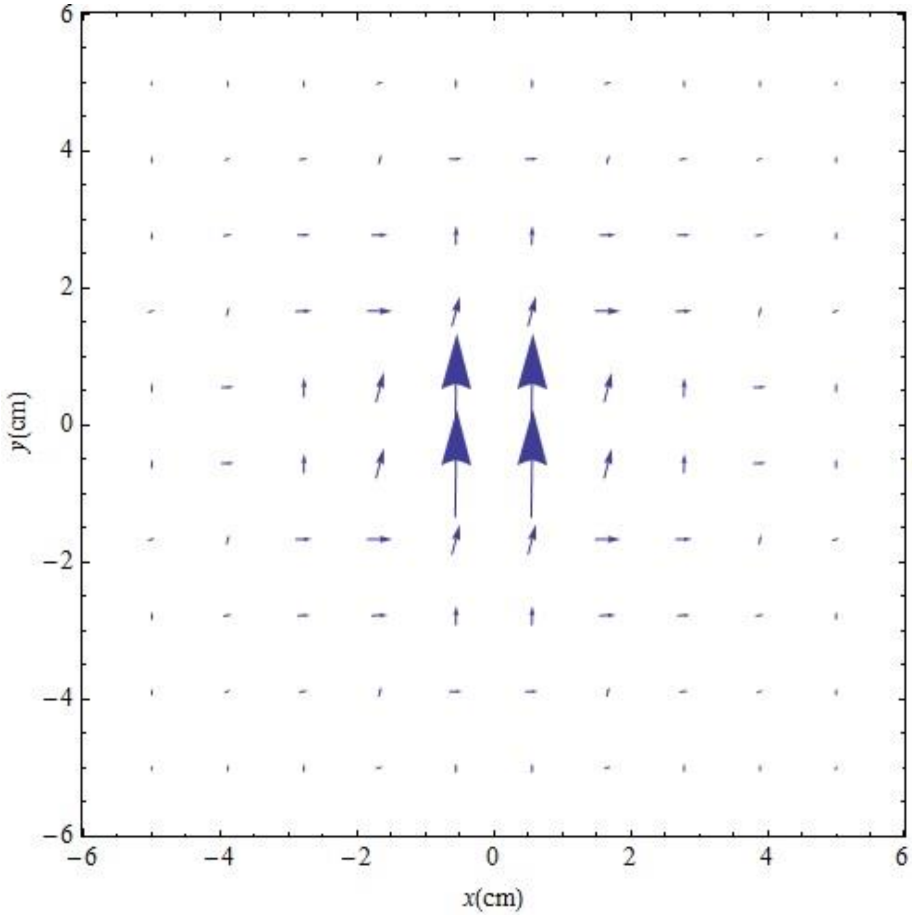

Fig.4 Diagram of the vortex amplitude function for m=2.

CONCLUSION

- In the present work vortex solutions for the components A_x and A_y of the vector amplitude function $\vec{A}$ are found.
- The basic vector nonlinear amplitude equation is presented as a system of two scalar equations for x and y components of the amplitude function.
- The graphics of the obtained solutions for different values of the vortex parameters are presented.
- The value of parameter n determines the number of spirals observed in the profiles of the intensity components (A_x and A_y) of the vector amplitude function.

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